Students' Mathematical Representations in the Classroom using TLSOA Model

Janthira Sumrit[1], Sudatip Hancherngchai[2]

[1,2]Department of Mathematics Education, Faculty of Education,

Phuket Rajabhat University

E-mail: s6586000103@pkru.ac.th

Abstract

This study analyzed students' mathematical representations in classrooms using Transformative Lesson Study Incorporated with Open Approach (TLSOA). The participants were 20 second-grade students from schools affiliated with the Phuket Provincial Administrative Organization. Data was gathered throughout the second semester of the 2024 school year using video cameras, still cameras, audio recorders, and field notes. Protocol and descriptive analyses were used to examine the data in accordance with Goldin's theoretical framework. The results showed that when employing TLSOA in the classroom, students' mathematical representations comprised three phases: (1) Collaboratively plan, where the lesson study team jointly analyzed mathematical concepts and designed problem situations; (2) Collaboratively do, where students represented mathematical concepts through gestures, written symbols, and verbal explanations; and (3) Collaboratively see, where the lesson study team jointly analyzed mathematical ideas from representations and proposed improvements. Students demonstrated understanding through counting, repeated addition, and multiplication concepts. These representations were evident in students' gestures, writing, and speech, showing progression from concrete counting to abstract multiplication understanding.

Keywords: Mathematical Representation, Lesson Study, Open Approach, TLSOA Model
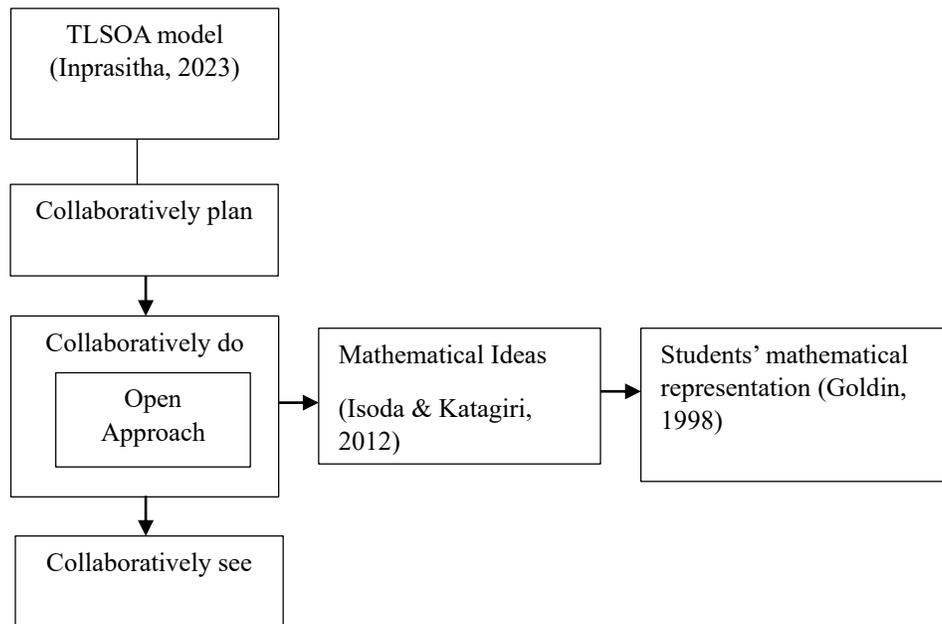
Introduction

One of the most important steps in learning mathematics is mathematical representation. Assist educators and learners in comprehending mathematical concepts. Therefore, in order to properly teach mathematics, teachers must comprehend student representation. (NCTM, 2000; Goldin & Shteingold, 2001). According to a number of studies in mathematics education, students' mathematical representations can be used to assess how well they understand mathematical ideas. These representations are observable and visible products, including mathematical equations, graphs, diagrams, and symbols. (Kaput, 1987; Duval, 2006; Goldin, 2014). As a result, representation is the foundation of learning mathematics and serves as a means of communicating mathematical ideas. A suitable learning environment is necessary for students to be able to produce and utilize meaningful representations, particularly by allowing them to confront challenges and find solutions on their own. (Lesh, Post, & Behr, 1987; Goldin, 2003).

An engaging teaching style that highlights student ideas and gives students the freedom to solve problems on their own in an unrestricted way. In the open approach, the problems used are incomplete or open-ended and presented in the first part of the lesson. The next step is to use a range of responses, which helps pupils get ready for the experience of learning new topics. Students' prior knowledge, abilities, or ways of thinking may be included into the teaching process (Becker & Shimada, 1997; Inprasitha, 2022, 2023). This kind of teaching places a strong emphasis on problem-solving. The problem-solving approach is more than just a method of instruction; it aims to help students become self-directed learners. The instructor has provided problem situations and tasks that are appropriate (Isoda & Katagiri, 2012; Takahashi, 2021). However, Thai teachers find open-ended problem design challenging because they are used to giving exercises and providing information and examples. Therefore, it is essential to have educational innovations like lesson study as another practice for instructors until it becomes a culture in the classroom. (Inprasitha, 2022; 2023). As a result, Lesson Study is a system of professional development for teachers in Japanese schools. The primary feature is that Japanese teachers gather in groups on a regular basis. to work together to construct lesson plans, develop innovative teaching strategies, test these strategies in actual classrooms, and refine them. The fundamental tenet of such a system is that creating and refining lessons inside the actual classroom setting is the most efficient approach to enhance and advance classroom instruction (Baba, 2007; Isoda & Katagiri, 2012; Isoda & Olfos, 2021). Since 2006, Inprasitha (2022) selected certain elements of the open approach and lesson study that have been implemented in Thai schools. Three essential steps—collaborative lesson plans, collaborative observation, and collaborative reflection—were incorporated into the lesson study process. The open approach used in the second stage has evolved into the TLSOA concept.

Classrooms using TLSOA model developed students' mathematical ideas (Inprasitha, 2023). As a result, it is a suitable setting for examining pupils' mathematical representations. Only the kinds or properties of mathematical representations have been examined in earlier studies. (Chaimontree, 2018; Chaiyawut et al., 2017; Khonkan, 2024; Monthaisong et al., 2023), and student representation in the flow of lessons (Chaiyaphruektol, 2018; Phosena & Thinwiangthong, 2023; Wadeesirisak & Pattanjak, 2022). The mathematical representation of mathematical concepts is still an unexplored research topic. Therefore, the purpose of this study was to analyze the students' mathematical representation based on their mathematical concepts that occurred in the classroom use the TLSOA model.

Conceptual Framework

```
┌─────────────────────┐
│   TLSOA model       │
│ (Inprasitha, 2023)  │
└─────────────────────┘
           │
┌─────────────────────┐
│ Collaboratively plan│
└─────────────────────┘
           │
           ▼
┌─────────────────────┐      ┌─────────────────────┐      ┌─────────────────────────┐
│ Collaboratively do  │      │ Mathematical Ideas  │      │ Students' mathematical  │
│  ┌───────────────┐  │─────▶│                     │─────▶│ representation (Goldin, │
│  │     Open      │  │      │ (Isoda & Katagiri,  │      │ 1998)                   │
│  │   Approach    │  │      │ 2012)               │      │                         │
│  └───────────────┘  │      └─────────────────────┘      └─────────────────────────┘
└─────────────────────┘
           │
           ▼
┌─────────────────────┐
│ Collaboratively see │
└─────────────────────┘
```

Figure 1: Conceptual Framework

Research objective

To analyze students' mathematical representations in classrooms using TLSOA model.

Research Methodology

Researchers, instructors, observers, and specialists worked together to create lesson plans for this study, which was carried out in a classroom setting utilizing the TLSOA methodology. Twenty second graders from PPAO Ban Talat Nuea School (Wan Kru 2502), Phuket Province, made up the target group for this study. The selection procedure was deliberate throughout the second semester of the 2024 academic year. The research tools are as follows:

1. Lesson plans. The following is how the lesson plans were created: 1) examined the Thai core curriculum's criteria and indicators, 2) chose activities from Gakko Tosho Publishing's mathematics textbooks in Japan, 3) create open-ended problem situations and instructional materials; 4) anticipate students' mathematical ideas; 5) set up the teaching sequence in accordance with the Open Approach; 6) put the lesson plans into practice in the classroom; and 7) evaluate the teaching outcomes and make improvements to the lesson plans for the following period.

Figure 2: Process of designing and developing the lesson plans.

2. Field notes documented the actions and words of the teacher and students in the classroom.

3. A video recorder was used to capture the teaching and learning process, particularly the ideas that students came up with in class.

4. An audio recorder is used to document the steps of lesson study not only during classroom instruction and learning, but also during the creation of lesson plans and after the lessons have been put into practice.

5. A camera was used to record lesson ideas, observe the classroom, and reflect on the lessons after they were taught.

6. Interview forms, students are interviewed using this form once the course is over.

Data collection: The researcher recorded audio and video, took photos, and took field notes in accordance with the lesson study procedures. Additionally, the researcher interviewed students about their thoughts following each hour of instruction.

Figure 3: Data collection according to the steps of the lesson study

Data Analysis: Students' worksheets, procedure, field notes, and interviews are examples of qualitative research data. A protocol analysis, content analysis, and analytical description based on the theoretical framework of mathematical representation (Goldin, 1998) were employed by the researchers.

Research Results

1. Collaboratively plan

The lesson study team collaborated to organize the teaching sequences, anticipate students' ideas, and determine the mathematical ideas of the lesson by analyzing the mathematics textbook. This lesson's

goal was to give students the ability to define multiplication as "the number of groups of objects with equal members in each group."

Expert 1	:	As mentioned below, the book notes suggest using symbolic words on page 10, The child *wrote that each box held eight chocolates, with three boxes holding eight pieces of peace*. He can't calculate, but when he calculates, he still *uses repeated addition* and then summarizes it. This is how we would express the computation as multiplication if we wrote it this way.
Expert 2	:	Finding what is left, determining how to arrange it equally, and determining the appropriate language to use to describe it are the main objectives of the class; the sentence is crucial, which is why they have colored it. You have to be prepared … *An explanation of the amount in every plate, box, and it must be unique for every child; what matters to them is*… *His writing, "How many plates are there?" is a statement that he finds meaningful. How is he going to write? He writes about a mathematical idea regarding the quantity of groups ...*

The lesson study team predicts the concepts that will arise during the collaboratively plan phase. They begin with counting whole numbers, explain them using repeated addition, and then go on to multiplication. Experts who highlight the idea of teaching multiplication through repeated addition, the significance of sentence form, and grouping are consulted when determining the number of groups and group members.

2. Collaboratively do

The lesson study team now implements collaborative lesson ideas in the classroom. The researcher served as a teaching observer and one of the team members served as a teacher. Let us engage in some shopping. The problem scenario is first presented by the teacher. "There are a lot of goods available. Look for the total amount of each item." Next, display the instructions and post images of the media, such as chocolate, fish, jelly, and pear, on the board. "Have students write a description of the quantity of each product." Students work independently to solve challenges. The teacher keeps track of the students' developing ideas. Next, bring those ideas together to discuss and compile all of the thoughts that have emerged in the classroom.

The mathematical concepts of the students in the Let's Go Shopping activity are as follows:

1) **Idea of Algorithms** in order to determine the total number of objects, pupils employ the process of counting one by one, which is represented by the following concepts:

Figure 4: Represents the concept of count all. With gestures and writing process to find results

A mathematical representation using gestures can be shown in figure 4. Students count from left to right while pointing with their index finger to each image on the worksheet. The final image represents the overall number, which is the outcome, till all the images are finished. Write down the total number of objects, as indicated on the worksheet. "24 pieces of Dubai chocolates" "Twelve fish are there." Showcase the pupils' understanding of results

Item 66	:	T	Okay, let's take a look at the S .7How does the S 7explain it? Let's listen to the S 7first.
Item 67	:	S7	*There are 24pieces of chocolate.*
Item 68	:	T	S7 said there are 24 pieces of chocolate on 3 plates.

It was discovered that students gave verbal explanations of topics from Protocol Items 66–88. Chocolate total: "There are 24 pieces of chocolate." It is evident from the speech that the pupils comprehend the outcome that the total number of members represents.

2) **Idea of Representation** the purpose of this concept is to express the number of groups in terms of multiplication. By determining that each group has the same number of members, students may comprehend the meaning of multiplication. By adding the number of members in each group, which symbolizes the opinions of the students, as follows:
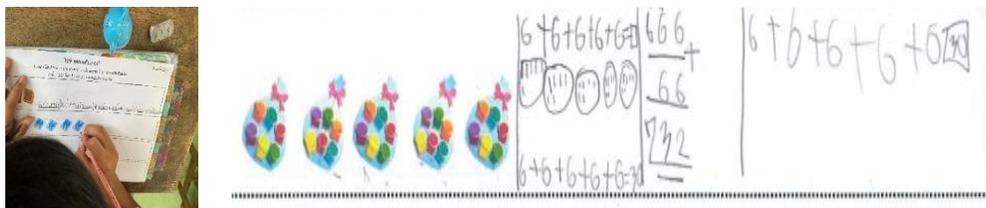
Figure 5: Represents the concept of repeated addition. With gestures and writing process to find results

Initially, students expressed their thinking by adding equal quantities repeatedly (6 + 6 + 6 + 6 + 6). While working on the task, students used gestures, drawings, and written expressions to organize the situation into equal groups. Through counting the number of groups and identifying the number of members

in each group, students gradually recognized the structure of equal groups. The use of gestures and visual representations supported students in coordinating the number of groups with the number of members in each group. Without explicit instruction or questioning from the teacher, students reorganized their thinking from repeated addition toward multiplicative reasoning. As a result, students began to interpret the expression 6 + 6 + 6 + 6 + 6 as 5 groups of 6, which reflects an emerging understanding of multiplication as a representation of equal groups.


Figure 6: Represented the idea of representation

Students create numbers to represent the number of groups based on Figure 6. 1–2–3–4–5–6 Next, construct language sentences and 2+2+2+2+2+2 =. "There are two fish in each of the six packs, for a total of twelve fish." It is evident that students use both mathematical symbols to express mathematical ideas. and the linguistic phrases that convey the concept of multiplication.

| Item 163 | : | T | Next, let's see what S4 says. What does S4 say? Let me read it to you. S4 say _there are 3 boxes of Dubai chocolates, and each box has 8 pieces of chocolate. How many are there in total?_ S4 Tell me, how many pieces do you have in total? |
| Item 164 | : | S4 | _24 pieces are obtained by adding them together 3 times._ |

Students describe the idea from protocol items 163–164 by stating how many groups there are and how many members are in each group. *"There are three boxes, with eight pieces of Dubai chocolates in each, for a total of twenty-four pieces."* obtained by summing them three times. Students demonstrate how to execute repeated additions to determine the outcome while knowing the number of members and groups.

3) Idea of Sets Students display how many members are in each group. Each group must have an equal number of members. As a result, it can be expressed as a multiplication. The following mathematical representations of the students' concepts were discovered:
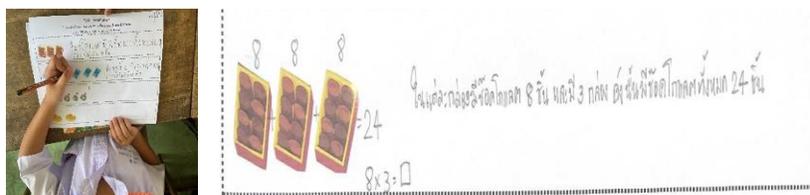
Figure 7: Represents the idea of Sets.

Figure 7 shows students using pencils to represent mathematical concepts through gestures. On the worksheet, indicate each group element with a point. Next, assign a number to each category. It is clear that instead of counting every photo, the pupils counted the individuals in groups. The exercise demonstrates how the students create symbolic statements and sentences that convey the idea of multiplication. "*There is a total of 24 chocolates because each of the three boxes contains eight chocolates.*" Additionally, write the multiplication sign phrase "*$8 \times 3 = \square$.*" Show that pupils are aware of the multiplication structure.

Item 165	:	T	Okay, let's listen to S 2first.
Item 166	:	S2	*Each box contains 8pieces of chocolate.*
Item 167	:	T	S2 said that there are 8 chocolates in each box. Look at S2. There are 8 chocolates in each box.
Item 168	:	S2	*And then there are 3 boxes.*

Students verbally state the number of individuals in each group and the total number of groups in order to illustrate the concept from protocol Items 165-168. Each box contains eight chocolates. "*Three boxes.*" Show that you grasp the multiplication structure. Students give examples of how to multiply to get the desired outcome.

3. Collaboratively see

The "Let us Go to the Market" activity's educational outcomes were considered by the lesson study team. As students' mathematical ideas emerge in the classroom, work together to reflect on and analyze them.
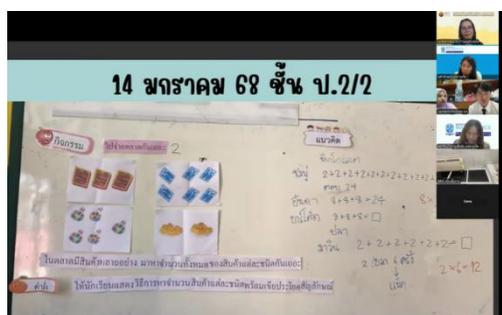

Figure 8: lesson study team reflects on lessons.

Teacher	:	We acquire the idea of chocolate from the instruction. According to *S7, there are twenty-four chocolates.* According to *S2, there are three boxes with eight chocolates each, for a total of twenty-four chocolates.* According to S4, *there are three boxes with eight pieces of chocolate in each, for a total of twenty-four pieces.* S2 then stated that *we must first examine the container's contents because the contents need to be combined. It is expressed as simply* the total for S7.

Observer : Today, a student saw a group of objects and wrote a symbolic sentence. _S2, he expressed it as a multiplication._ However, the majority still _write addition like in S4, adding repeatedly, and some count like in S7_. However, _some people write both addition and multiplication._ It was agreed that if it were multiplication, the contents of the containers had to be written first. They would emphasize the containers first and then count how many there were.

Expert 1 : As a result, there are many different ways to find the quantity of products and compose a symbolic phrase. _He obeyed commands and used addition and multiplication symbols in a number of statements, which is perfectly acceptable._ Because he was asked to find and write symbolic sentences, he found them by _adding, multiplying, or counting_.

Expert 2 : However, none of them are on board, correct? Therefore, _8+8 is correct as he followed the instructions when writing the symbolic sentence._ However, the worksheet specifies that it must only be written as a multiplication symbol sentence in the condition. Therefore, it is doubtful that a child wrote this accurately. _He used symbolic language to write it in a meaningful way._

Expert 3 : _According to what I have heard from the children, the concept is at a level where discussion is possible._ What did we say yesterday? What kind of green is it? You must determine whether the green hue is the same in the container and on the plate. After that, you may determine how many boxes of pink and bits of green are in the container. You can now proceed from this point when crafting a symbolic sentence.

From the reflection protocol, Both the teacher and the observer considered the mathematical ideas that the students had developed. They discovered the concepts of multiplication, repeated addition, and counting of all. These concepts were seen to be represented by the students' gestures, written descriptions of the process or result, and spoken multiplication sign language. Furthermore, the idea of analyzing the representation that occurred this time was confirmed by experts, along with suggestions for developing mathematical concepts in the upcoming period.

Conclusion

Using the TLSOA model in the classroom, it was discovered that the first phase, cooperatively planned, involved the lesson study team analyzing the mathematics textbooks to identify the mathematical concepts of the lessons, create a teaching sequence, and anticipate student ideas. In the second step, collaboratively do, students solve mathematical problems on their own and learn mathematical concepts through repetitive addition, multiplication, and counting. Students' mathematical representations in speech, writing, and gestures make these concepts clear. The third step, collaboratively see; in order to identify the students' mathematical concepts, the lesson study team reflects on and analyzes their mathematical

representations. They then collaboratively suggest strategies to develop ideas for the following period. Additionally, employing the TLSOA model in the classroom helps students develop their mathematical concepts because they may answer the problems on their own and share them with others through written, spoken, and gestural representations of Goldin's ideas.

Research suggestions

1. Suggestions for applying research results.

Although this study was conducted in a TLSOA model classroom, the findings can be applied to general mathematics classrooms beyond the TLSOA context. Teachers can emphasize students' mathematical representations, such as gestures, drawings, and written expressions, to support students' understanding of multiplication. Allowing students to begin with repeated addition and observing how they organize equal groups may help support the transition from additive to multiplicative reasoning without direct procedural instruction.

2. Suggestions for future research.

Future research should examine students' mathematical representations in the affective domain, including confidence, engagement, and attitudes toward learning mathematics, to gain deeper insights into students' learning processes.

References

Baba, T. (2007). How is lesson study implemented? In M. Isoda, M. Stephens, Y. Ohara, & T. Miyakawa (Eds.), Japanese lesson study in mathematics: Its impact, diversity and potential for educational improvement (pp. 2–7). World Scientific. https://doi.org/10.1142/9789812707475_0001

Becker, J. P., & Shimada, S. (Eds.). (1997). *The open-ended approach: A new proposal for teaching mathematics.* National Council of Teachers of Mathematics.

Chaiyawut, J., Sichomphu, S., & Inprasitha, M. (2017). A survey of students' mathematical-understanding representations in Grade 11 open-approach classrooms. *KKU Research Journal (Graduate Studies) – Humanities and Social Sciences*, 5(2), 56–67.

Chaimontree, S. (2018). *A study of representation of fraction of grade seven students* [Unpublished master's thesis]. Rajabhat Maha Sarakham University.

Chaiyaphruektol, W. (2018, March 9). *Representations in flow of lesson in classroom using lesson study and open approach.* Graduate School, Khon Kaen University. https://app.gs.kku.ac.th/gs/th/publicationfile/item/19th-ngrc-2018/HMP5/HMP5.pdf

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics, 61*(1–2), 103–131. https://doi.org/10.1007/s10649-006-0400-z

Goldin, G. A. (1998). Representational systems, learning, and problem solving in mathematics. *Journal of Mathematical Behavior, 17*(2), 137–165.

Goldin, G. A. (2003). Representation in school mathematics: A unifying research perspective. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to Principles and Standards for School Mathematics* (pp. 275–285). National Council of Teachers of Mathematics.

Goldin, G. A. (2014). Representation in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 531–536). Springer.

Goldin, G. A., & Shteingold, N. (2001). Systems of representation and the development of mathematical concepts. In A. A. Cuoco & F. R. Curcio (Eds.), *The roles of representation in school mathematics* (pp. 1–23). National Council of Teachers of Mathematics.

Inprasitha, M. (2022). Lesson study and open approach development in Thailand: A longitudinal study. *International Journal for Lesson and Learning Studies, 11*(5), 1–15. https://doi.org/10.1108/IJLLS-04-2021-0029

Inprasitha, M. (2023). Blended learning classroom model: A new extended teaching approach for new normal. *International Journal for Lesson and Learning Studies, 12*(4), 288–300. https://doi.org/10.1108/IJLLS-01-2023-0011

Isoda, M., & Katagiri, S. (2012). *Mathematical thinking: How to develop it in the classroom.* World Scientific. https://doi.org/10.1142/8163

Isoda, M., & Olfos, R. (Eds.). (2021). *Teaching multiplication with lesson study: Japanese and Ibero-American theories for international mathematics education.* Springer. https://doi.org/10.1007/978-3-030-28561-6

Kaput, J. J. (1987). Representation systems and mathematics. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 19–26). Lawrence Erlbaum Associates.

Khonkan, R. (2024). Students' mathematical representations of multiplication in lesson-study/open-approach classrooms. *Valaya Alongkorn Rajabhat University Graduate Research Journal*, 18(1), 147–160.

Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 33–40). Lawrence Erlbaum Associates.

Monthaisong, K., Pavaputanon, L., & Boonsena, N. (2023). Connections of students' mathematical representations in problem-solving classroom. *KKU Research Journal (Graduate Studies), 23*(1), 107–118. https://ph02.tcithaijo.org/index.php/gskku/article/download/246125/168564

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* National Council of Teachers of Mathematics.

Phosena, M., & Thinwiangthong, S. (2023). Students' representations across instructional sequences in open-approach classrooms. *The Journal of Research and Academics*, 6(1), 63-78.

Takahashi, A. (2021). *Teaching mathematics through problem-solving: A pedagogical approach from Japan.* Routledge. https://doi.org/10.4324/9781003015475

Wadeesirisak, T., & Pattanjak, A. (2022). An analysis of student's mathematical understanding through flow of lesson in open-ended problem solving. *Journal of MCU Social Science Review,* 11(6), R140–R154. https://so03.tci-thaijo.org/index.php/jssr/article/view/255612